Principles of Radio Astronomy and Ground Based CMB Detection

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Abstract

After an introduction about common measures of electromagnetic radiation and the nature of blackbody radiation, the basic concepts of radio astronomy are explained. Next, an overview of emission sources is presented, including an account of the discovery and origin of the cosmic microwave background (CMB). Finally, the prerequisites for detecting the CMB radiation from Earth's surface are discussed.

Contents

1 Measures of electromagnetic radiation

1.1 Luminosity

The luminosity L of an object is the power (or energy per unit of time) emitted in all directions and over the entire electromagnetic frequency spectrum. The total energy E emitted by the object within a certain period of time is then the time integral of the luminosity:

$$E = \int L \, dt$$

It is practically impossible to measure the luminosity of a celestial source directly, given that it is impossible to make measurements in all directions around it and given that all practical receiver systems have a limited bandwidth.

1.2 Flux

To eliminate the need to measure in all directions around the source, the flux F is defined as the received power per unit area. Assuming that the source radiates isotropically, i.e. equal amounts of energy in all directions, and taking the formula for the surface area of a sphere with radius d into account, the flux at a distance d from the source is:

$$F = \frac{L}{4 \, \pi \, d^2}$$

Using the flux F as parameter, the total emitted energy E is given by:

$$E = \iint F \, dA \, dt$$

1.3 Flux density

Receiver systems always operate over a certain limited bandwidth $\Delta\nu$, never over the entire frequency spectrum. The flux density F_ν is therefore a more practical measure. It is defined as the flux per unit of frequency:

$$F_\nu = \frac{F}{\Delta\nu}$$

Using the flux density F_ν as parameter, the total emitted energy E is given by:

$$E = \iiint F_\nu \, d\nu \, dA \, dt$$

In radio astronomy applications, flux densities are usually very small which explains why radio astronomers have defined a special purpose unit for it named after Karl Jansky, an American physicist and radio engineer working at Bell Telephone Laboratories and generally regarded as the father of radio astronomy:

$$1 \text{ Jy} = 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}$$

1.4 Surface brightness

So far, it has been assumed that the source is a point-like source which is not necessarily the case. The surface brightness (intensity) of an object is defined as the flux density F_ν per unit solid angle Ω subtended by the object:

$$B_\nu = \frac{F_\nu}{\Omega}$$

Surface brightness (intensity) is independent of distance as both the flux density F_ν and the solid angle Ω are inversely proportional to the square of the distance, thereby cancelling-out the distance dependency.

2 Blackbody radiation

2.1 Planck's law

Imagine a completely opaque object which does not reflect any light, i.e. a perfect absorber. In the absence of reflections, one could think of such an object as being ”black”. However, if internally matter and light freely and abundantly interact and are able to settle into a state of thermal equilibrium, the object emits electromagnetic radiation with a particular spectrum which is only dependent on its thermodynamic temperature T.

The surface brightness B_ν of the resulting blackbody radiation as a function of frequency ν and thermodynamic temperature T is given by Planck's law, with c the speed of light, h Planck's constant and k_B Boltzmann's constant:

$$B_\nu = \frac{2\,h\,\nu^3}{c^2}\,\frac{1}{e^{h\,\nu/k_B\,T}-1}$$

The resulting spectrum for different temperatures is shown in figure 1. The peaks of the curves are located at frequencies ν which solely depend on the thermodynamic temperature T:

$$\nu_{peak} = 5.879 \times 10^{10}\;\; T \tag{1}$$

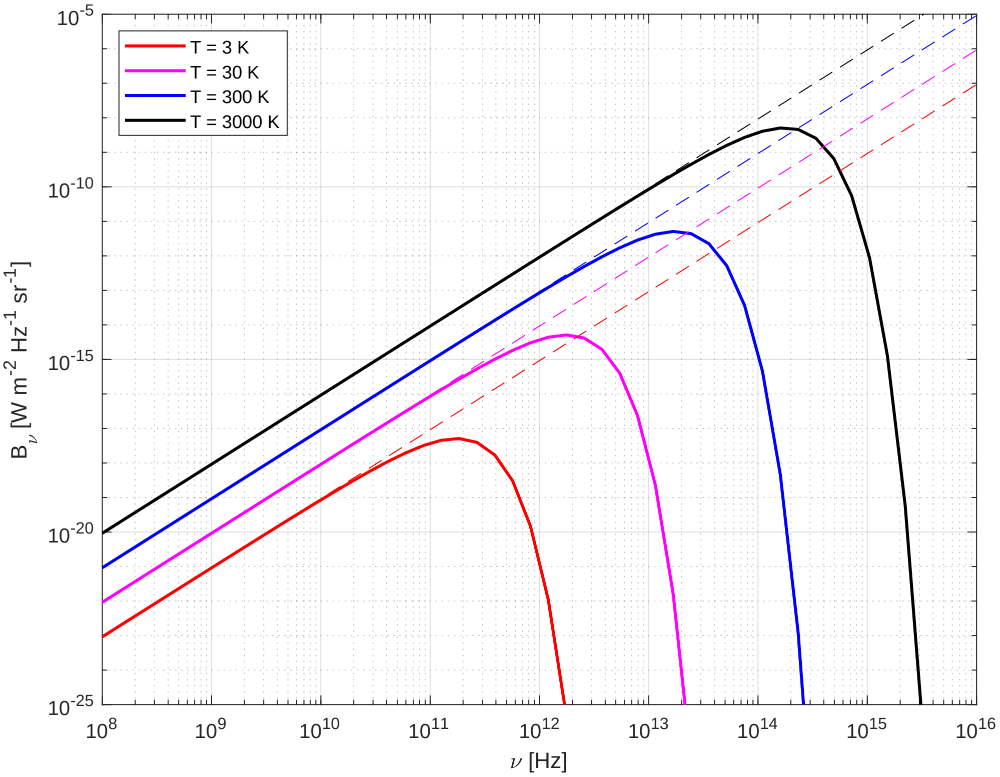

Figure 1: Blackbody spectrum for different temperatures according Planck's law. The dashed lines represent the respective Rayleigh-Jeans approximations.

2.2 Rayleigh-Jeans' law

In cases where $h\,\nu/k_B\,T \ll 1$, Planck's law is approximated to a reasonable extent by what is known as Rayleigh-Jeans' law, expressible either as a function of frequency ν or as a function of wavelength λ:

$$B_\nu = \frac{2\,k_B\,\nu^2}{c^2}\,T$$

$$B_\nu = \frac{2\,k_B}{\lambda^2}\,T \qquad (2)$$

The Rayleigh-Jeans approximations of the blackbody spectra in figure 1 are represented by dashed lines and are clearly only valid at the lower frequency end.

2.3 Brightness temperature

The Rayleigh-Jeans approximation allows quantifying the surface brightness B_ν of a source as an equivalent brightness temperature T_B:

$$\boxed{T_B = \frac{c^2}{2\,k_B\,\nu^2}\,B_\nu}$$

In cases where Rayleigh-Jeans' law is not a valid approximation, the brightness temperature T_B of a source has to be defined using Planck's law.

As an illustration, Earth's atmosphere has a surface brightness of $2 \times 10^{-15}\ \mathrm{Wm^{-2}Hz^{-1}sr^{-1}}$ at $350\ \mathrm{GHz}$ which corresponds to a brightness temperature of $52\ \mathrm{K}$. At $30\ \mathrm{GHz}$, it has a surface brightness of $2 \times 10^{-18}\ \mathrm{Wm^{-2}Hz^{-1}sr^{-1}}$ which corresponds to a brightness temperature of $7\ \mathrm{K}$. The brightness temperature is clearly a function of frequency, the reason being that our atmosphere is not a blackbody. This leads to the conclusion:

> In general, brightness temperature is a measure of surface brightness (intensity) and is a property of the radiation, not of the emitting object.

Only when the source is opaque and thermal, its brightness temperature corresponds to its thermodynamic temperature and is independent of frequency.

3 Principles of radio astronomy

3.1 Components of a radio telescope

Radio astronomers observe the sky in a part of the electromagnetic spectrum invisible to the human eye with frequencies ranging from broadly $10\ \mathrm{MHz}$ to $100\ \mathrm{GHz}$. Radio telescopes are often recognizable by their large dish antennas, needed to achieve sufficient gain and resolution. As such, they more resemble enlarged satellite TV systems than the optical instruments used by visible light astronomers.

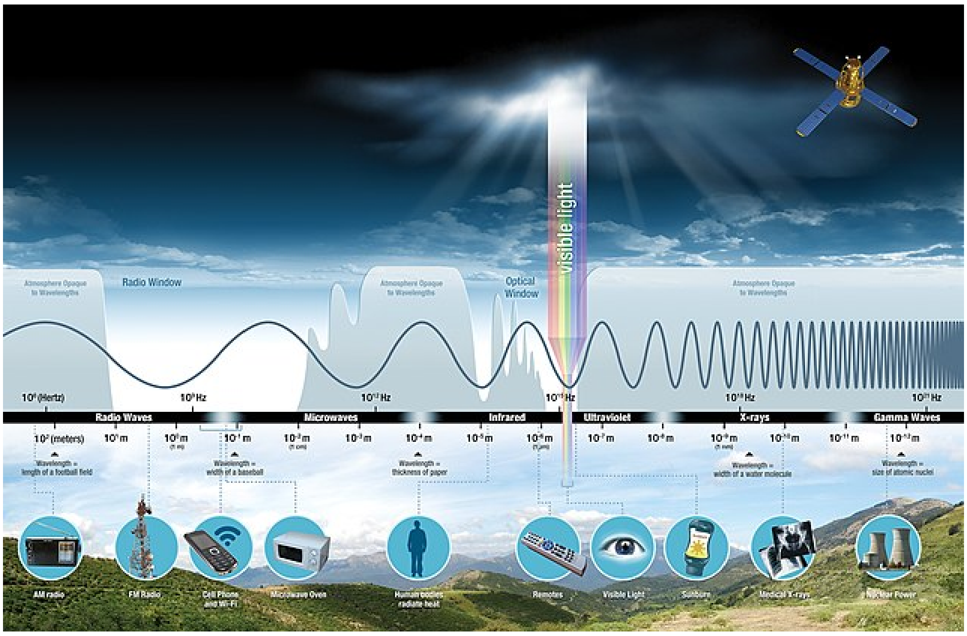

Figure 2: The electromagnetic spectrum ranging from gamma rays to radio waves. Earth's atmosphere is opaque for the larger part, with the exception of a "radio window" and an "optical window", allowing radio and optical telescopes to make ground based observations. (Credit: NASA)

Many receiver systems used in radio astronomy applications are superheterodyne receivers. They convert a high frequency signal to an equivalent lower frequency signal which is easier to amplify, transmit and process without significant losses and without adding large amounts of noise to the signal. The block diagram of a typical superheterodyne receiver is shown in figure 3.

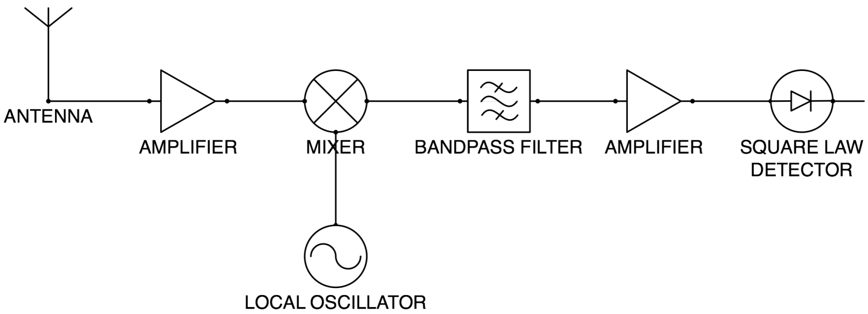

Figure 3: Main components of a superheterodyne receiver. Key principle is the mixing of the antenna signal with the output of a local oscillator to achieve a down-conversion to a lower frequency range.

The signal picked-up by the antenna is pre-amplified and then mixed with the signal from a local oscillator operating at a frequency slightly lower than the frequency of observation. The mixer outputs a signal which contains the sum and difference frequencies of the antenna signal and the local oscillator. The bandpass filter then eliminates everything except a defined bandwidth around the difference frequency. The output of the bandpass filter is essentially the same as what the antenna picked-up but centered around a much lower and easier to process frequency. It is amplified and subsequently converted by a square law detector (usually a power diode) into a voltage which is proportional to the received power.

3.2 Antenna temperature

3.2.1 Nyquist's theorem

Harry Nyquist, a Swedish-born American electronics engineer at Bell Telephone Laboratories, has shown in 1928 that a resistor in an electronic circuit creates noise with a power per unit bandwidth which is solely dependent on the resistor's thermodynamic temperature. In general, the power in an electronic circuit is therefore often expressed as an equivalent temperature T_{eq}, namely the thermodynamic temperature a resistor would need to have to generate the same (noise) power level:

$$T_{eq} = \frac{P}{k_B \, \Delta\nu} \tag{3}$$

Alternatively:

$$P = k_B \, \Delta\nu \, T_{eq} \tag{4}$$

3.2.2 Definition

If A_{eff} is the effective area of the antenna of a receiver system with bandwidth $\Delta\nu$, the total power P collected by the antenna from the source under observation is given by [MSK16, p. 144]:

$$P = \frac{1}{2} \left(\iint B_\nu(\theta, \phi) \, P_n(\theta, \phi) \, sin \, \theta \, d\theta \, d\phi \right) \Delta\nu \, A_{eff} \tag{5}$$

$B_\nu(\theta, \phi)$ is the surface brightness of the source as a function of 2 sky coordinates θ and ϕ and $P_n(\theta, \phi)$ is the normalized power pattern of the antenna as a function of the same, usually angular, coordinates. An antenna's power pattern $P(\theta, \phi)$ describes its radiation pattern in absolute values while the normalized power pattern $P_n(\theta, \phi)$ describes the same as a fraction of the peak power. In other words, the normalized power pattern equals the power pattern divided by the peak value and therefore varies between 0 and 1. The factor $1/2$ in equation (5) accounts for the practical restriction that an antenna is usually only sensitive for either horizontally or vertically polarized waves and not for both, which means that it only captures half of the power from a randomly polarized source.

The equivalent temperature of the power collected by the antenna from the source under observation is commonly called the antenna temperature T_A. Combining equations (3) and (5) with $T_{eq} = T_A$ gives:

$$T_A = \frac{A_{eff}}{2 \, k_B} \iint B_\nu(\theta, \phi) \, P_n(\theta, \phi) \, sin \, \theta \, d\theta \, d\phi \tag{6}$$

3.2.3 Sources uniformly filling the sky

Using the Rayleigh-Jeans approximation (2), the surface brightness $B_\nu(\theta, \phi)$ of a source as a function of its brightness temperature $T_B(\theta, \phi)$ is given by [MSK16, p. 149]:

$$B_\nu(\theta, \phi) = \frac{2\,k_B}{\lambda^2}\,T_B(\theta, \phi)$$

For a source uniformly filling the sky, $T_B(\theta, \phi)$ is a constant. This allows moving the entire expression for $B_\nu(\theta, \phi)$ out of the integral in equation (6):

$$T_A = \frac{A_{eff}}{\lambda^2}\,T_B \underbrace{\iint P_n(\theta, \phi)\,sin\,\theta\,d\theta\,d\phi}_{\Omega_A}$$

The integral is actually the solid angle Ω_A subtended by the antenna.

The antenna theorem states that the product of the solid angle Ω_A and the antenna's effective area A_{eff} equals the square of the wavelength λ:

$$\Omega_A\,A_{eff} = \lambda^2$$

Putting it all together shows that:

$$\boxed{T_A = T_B}$$

Or in plain words:

For a source uniformly filling the sky, the antenna temperature T_A is a direct measure of the source's brightness temperature T_B.

3.2.4 Point-like sources

When observing a point-like source, i.e. a source of which the solid angle is much smaller than the solid angle subtended by the antenna, the normalized power pattern of the antenna $P_n(\theta, \phi)$ practically equals unity over the entire solid angle of the source. Equation (6) then becomes [MSK16, p. 145]:

$$T_A = \frac{A_{eff}}{2\,k_B} \underbrace{\iint B_\nu(\theta, \phi)\,sin\,\theta\,d\theta\,d\phi}_{F_\nu}$$

The integral is actually the flux density F_ν and hence:

$$T_A = \frac{A_{eff}}{2\,k_B}\,F_\nu$$

Stated differently:

For a point like source, the antenna temperature T_A
is a direct measure of the source's flux density F_ν.

3.3 Amplification and detection

The power levels delivered to a receiver by an antenna are very small and would be hard to process without amplification. If G is the overall gain of a receiver's amplifier(s), the power P_{out} at the output of the amplification stage as a function of the input power P_{in} is given by:

$$P_{out} = G\,P_{in}$$

Taking equation (4) into account, this becomes:

$$P_{out} = G\,k_B\,\Delta\nu\,T_{eq}$$

The power detector of the receiver system converts power into a proportional voltage (subsequently digitized and recorded for further analysis). If the proportionality factor of the power detector is α, its output voltage V_{out} is given by:

$$V_{out} = \alpha\,P_{out}$$

$$V_{out} = \alpha\,G\,k_B\,\Delta\nu\,T_{eq}$$

The factor $\alpha G\,k_B\,\Delta\nu$, which is nothing more than a conversion factor between temperature and voltage, is not calculated by multiplying the individual subfactors but is determined as a whole via a calibration procedure. It is therefore convenient to introduce the overall conversion factor V_K as:

$$V_K = \alpha\,G\,k_B\,\Delta\nu$$

Part of the detected power originates from the source under observation but the majority is due to unwanted contributions. As mentioned before, the equivalent temperature of the source of interest is usually termed antenna temperature T_A. The sum of all unwanted contributions is often denoted as system temperature T_{sys}. The output voltage V_{out} of the power detector is then proportional to the sum of both:

$$V_{out} = V_K \, (T_A + T_{sys}) \tag{7}$$

3.4 System calibration

3.4.1 Purpose

To be able to derive the equivalent temperature of a measured output voltage, the proportionality factor V_K needs to be known. This is achieved by means of a calibration procedure. One method uses a noise diode to carry out the calibration, an alternative one makes use of a combination of a hot and cold load. The advantage of the latter method is that at the same time, the receiver noise T_{rcv} gets quantified.

3.4.2 Using a noise diode

The noise diode calibration process essentially consists of first taking a measurement excluding the source under observation but retaining all unwanted contributions. A known amount of noise is then injected into the receiver system, resulting in an output voltage increase. The ratio of the voltage increase versus the injected noise gives the proportionality factor.

To make the measurement which excludes the source under observation, i.e. the so called off-source observation, the antenna is pointed to a piece of the sky away from the source but still close enough to it so that e.g. the galactic and atmospheric contributions are not too different. When the telescope is looking at the ”empty” sky, equation (7) becomes:

$$V_{off} = V_K \, T_{sys} \tag{8}$$

While the antenna is still pointing at the same piece of ”empty” sky, a calibration noise diode is switched on, adding an amount of noise to the measurement with a known equivalent temperature T_{cal}:

$$V_{cal} = V_K \, (T_{sys} + T_{cal}) \tag{9}$$

Subtracting equation (8) from equation (9) gives:

$$V_{cal} - V_{off} = V_K \, T_{cal}$$

Reworking this equation yields:

$$\boxed{V_K = \frac{V_{cal} - V_{off}}{T_{cal}}} \tag{10}$$

3.4.3 Using hot and cold loads

When the antenna is completely covered by a source, the antenna temperature equals the brightness temperature of the source as explained before. For measurements V_H and V_L, respectively obtained with a source at a known higher temperature $T_A = T_H$ and a source at a known lower temperature $T_A = T_L$, equation (7) becomes:

$$V_H = V_K\,(T_H + T_{rcv}) \tag{11}$$

$$V_L = V_K\,(T_L + T_{rcv})$$

Subtracting both from each other gives:

$$V_H - V_L = V_K\,(T_H - T_L)$$

Reworking this equation yields:

$$\boxed{V_K = \frac{V_H - V_L}{T_H - T_L}} \tag{12}$$

3.5 Measurement procedure

The antenna temperature of the source under observation is determined by subtracting the off-source and on-source output voltages from each other and then converting the resulting voltage difference into an equivalent temperature. Equation (7) applies for the on-source measurement:

$$V_{on} = V_K\,(T_A + T_{sys}) \tag{13}$$

Subtracting equation (8) from equation (13) gives:

$$V_{on} - V_{off} = V_K\,T_A$$

Rearranging this equation yields:

$$\boxed{T_A = \frac{1}{V_K}\,(V_{on} - V_{off})}$$

3.6 Measurement uncertainty

In statistics, the standard deviation σ of a data set is a measure for the amount of variation among the values in the set. If a variable x takes random values $x_1, x_2, ..., x_N$ during N independent measurements, all with equal probability and with an average value $\overline{x}$, its standard deviation is given by:

$$\sigma(x) = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \overline{x})^2}{N}} \tag{14}$$

At radio frequencies, the variation in the measured power is proportional to the measured power in itself and consequently, the variation in the equivalent temperature proportional to the equivalent temperature per se. In radio astronomy applications, the antenna temperature T_A is usually much smaller than the system temperature T_{sys} and the uncertainty is therefore dominated by the fluctuations in the latter. Additionally, the number of independent measurements N for radio observations is given by $\Delta t\ \Delta\nu$ where Δt is the integration time and $\Delta\nu$ the receiver bandwidth. Equation (14) therefore translates into [MSK16, p. 132]:

$$\sigma(T_A) \approx \frac{T_{sys}}{\sqrt{\Delta t\ \Delta\nu}}$$

To determine the antenna temperature T_A, an off-source observation is subtracted from an on-source observation. Both observations are made independently from each other which means that the variance[1] of T_A is the sum of the variances of both observations. As both observations are subject to the same variance:

$$\sigma^2(T_A) \approx 2\left(\frac{T_{sys}}{\sqrt{\Delta t\ \Delta\nu}}\right)^2$$

In terms of the standard deviation, this becomes:

$$\sigma(T_A) \approx \sqrt{2}\,\frac{T_{sys}}{\sqrt{\Delta t\ \Delta\nu}} \tag{15}$$

Equation (15) is an approximation because it does not take the uncertainty due to receiver gain fluctuations into account. It is however still useful to determine if a source is detectable within a given observation time or alternatively, to determine the minimum observation time needed to positively detect a source.

The faintest detectable signal is often expressed in terms of the signal to noise ratio SNR:

$$SNR = \frac{T_A}{\sigma(T_A)} \tag{16}$$

[1] The variance is the square of the standard deviation.

Equation (15) rewritten for Δt and combined with equation (16) gives:

$$\Delta t \approx \frac{2}{\Delta\nu}\left(SNR\,\frac{T_{sys}}{T_A}\right)^2$$

Usually, a source is considered positively detected when the $SNR \geq 3$. Note that Δt is the time needed for the on-source observation which equals the time needed for the off-source observation. The total observation time is consequently twice as long.

4 Emission sources

4.1 Discrete radio sources

Various discrete radio sources exist in the universe [Ver07]. Some are relatively nearby like the Solar system planets while others are located extremely far away like quasars but are still detectable from Earth. They are point-like sources for antennae with relatively large beam widths and contribute very little to the overall equivalent temperature given that they are rather weak.

The Solar system planets emit blackbody radiation as a result of their heating by the Sun. Jupiter additionally emits cyclotron as well as synchrotron radiation. Cyclotron radiation is caused by non-relativistic electrons spiralling in Jupiter's strong magnetic field near its magnetic poles (decametric emission). Synchrotron radiation originates from relativistic electrons spiralling in Jupiter's strong magnetic field near its magnetic equator (decimetric emission).

Inside our Milky Way galaxy, radio surveys have identified ”radio” stars which are generating peculiarly intense radio signals. But there are also micro-quasars, pulsars and other supernova remnants as well as emission nebulae, planetary nebulae and masers. Last but not least, the galactic center is known to be a strong radio source.

A micro-quasar is formed by a neutron star or black hole surrounded by material from a close binary companion in a so called accretion disk. The infalling matter is heated to the point that it produces X-rays, while radio jets appear to be ejected perpendicular to the accretion disk.

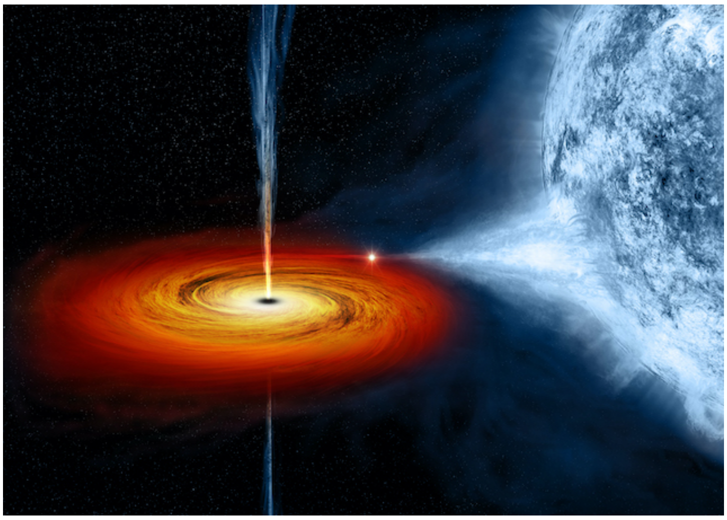

Figure 4: Artist impression of the Cygnus X-1 microquasar. Cygnus X-1 is a stellar-mass black hole which pulls material from a massive, blue companion star toward it. This material forms a disk (shown in red and orange) that rotates around the black hole before falling into it or being redirected away from the black hole in the form of powerful jets. (Credit: NASA/CXC/M.Weiss)

A pulsar is a highly magnetized neutron star that beams out radiation along its magnetic poles while spinning around its axis of rotation. This results in a highly stable repetitive signal like the beam of a light house sweeping by at regular intervals.

Emission nebulae or HII regions are clouds of ionized hydrogen. The ionization is caused by the energetic UV light from nearby young stars and results in charged protons and electrons freely wandering around and producing radio emissions.

A planetary nebula originates from the death of a less massive star like our Sun. In the process of becoming a white dwarf at the end of its lifetime, such a star expels its outer layers which form the nebula. The historical term ”planetary nebula” is somewhat misleading in the sense that it suggests that planets play an important role in the nebula's creation process, which is not the case.

Under certain conditions, molecular clouds in the universe emit far more energy in the microwave region of the electromagnetic spectrum than expected. One of the prerequisites is that energy is pumped into them by some external energy source, usually a nearby star. The result is a maser, named after the acronym for ”microwave amplification by stimulated emission of radiation”.

At extra-galactic level, a distinction was historically made between radio galaxies, quasars and Seyfert galaxies but all 3 are now understood to be variations of what are called active galactic nuclei (AGN). They are characterized, beside by a supermassive black hole at their center, by two narrow jets of material escaping in opposite directions and ending in giant lobes of radio emitting plasma. Radio galaxies are AGN with their jets directed across the sky as seen from Earth, while quasars have their jets directed towards us. Seyfert galaxies are AGN of which the jets have to make themselves a way out of the dusty or gaseous core of the galaxy.

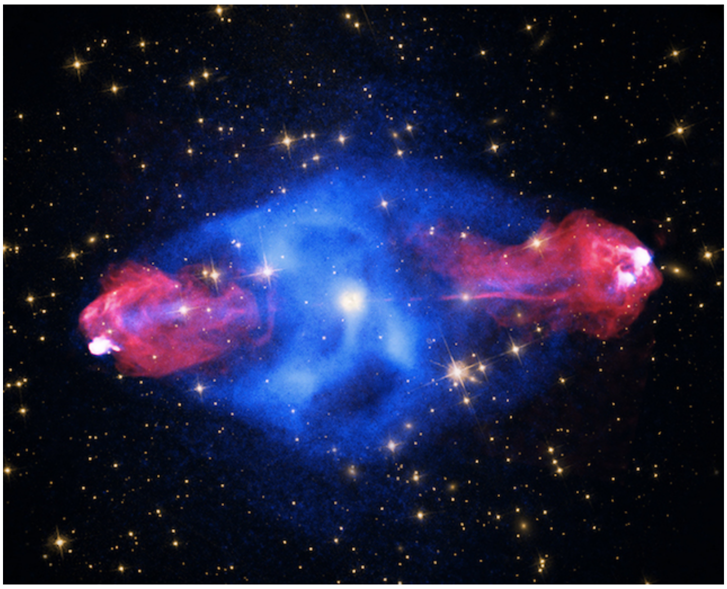

Figure 5: Composite image of Cygnus A, an AGN galaxy about 700 million light years away. The superposition consists of X-ray (blue), radio (red) and visible light (yellow) data. The radio image reveals ”hot spots” about 300000 light years out from the center of the galaxy where powerful jets emanating from the galaxy’s supermassive black hole end. (Credit: X-ray: NASA/CXC/SAO; Radio: NSF/NRAO/AUI/VLA; Optical: NASA/STScI)

4.2 Galactic emission

Extended galactic radio emissions mainly consist of synchrotron radiation and bremsstrahlung, as well as the $1420.405\ \mathrm{MHz}$ or $21\ \mathrm{cm}$ spin flip spectral line emission from cold neutral hydrogen HI. Synchrotron radiation is produced by relativistic electrons spiraling around magnetic field lines in the galaxy. Bremsstrahlung is created when electrons are deflected by positively charged particles, typically protons.

Figure 6 represents a radio map of the galaxy at a frequency of $44.1\ \mathrm{GHz}$. It shows that the galactic radiation is more pronounced around the galactic plane, which runs horizontally through the center of the figure, and is less at higher galactic latitudes. Interpolating the results of relevant surveys yields a reasonable estimate of the galactic contribution at a particular frequency.

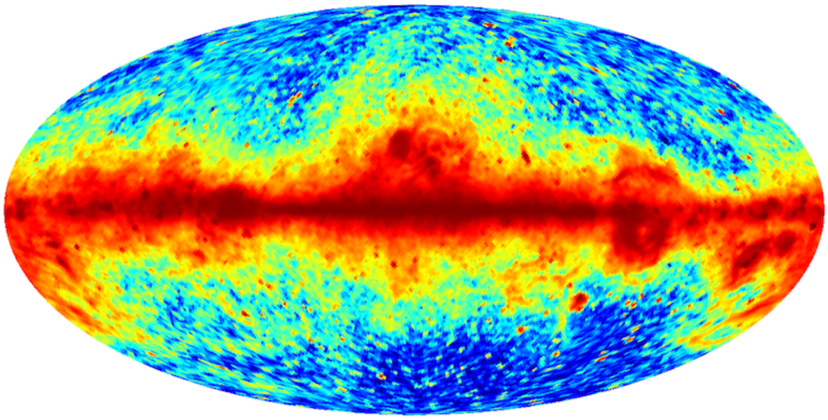

Figure 6: Galactic all-sky map at $44.1\ \mathrm{GHz}$ (with the cosmic microwave background subtracted) made by means of the Planck satellite. (Credit: ESA)

4.3 Cosmic microwave background

4.3.1 Discovery

In 1964, Arno Penzias and Robert Wilson, astronomers working at Bell Laboratories in Holmdel, New Jersey, were planning to study radio waves from the center of our galaxy. For that purpose, they used a steerable microwave horn antenna, previously employed to detect radio waves bounced-off by satellites. After subtracting the noise generated in their receiver system and the noise originating from Earth's atmosphere, Penzias and Wilson were left with an unexplainable hiss which turned out to be independent of antenna direction and invariant over time. They originally held the antenna responsible but after a thorough cleaning an eventually even the "permanent" removal of nesting pigeons, the unexplainable hiss remained present.

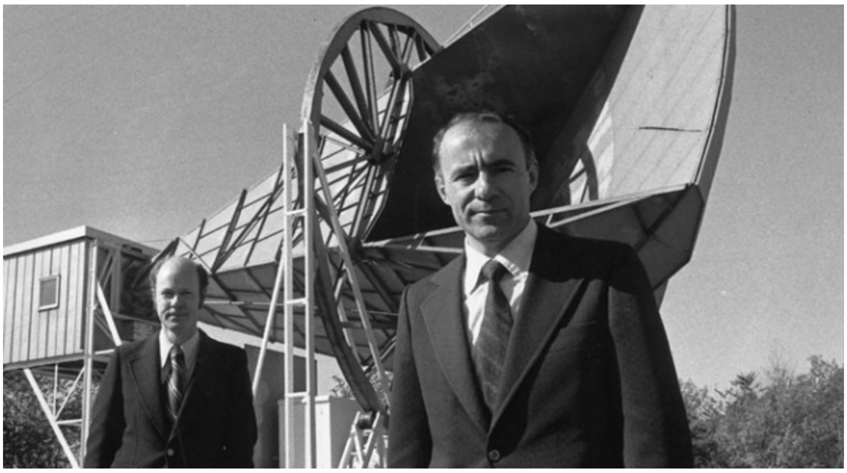

Figure 7: Robert Wilson and Arno Penzias in front of the horn antenna at Bell Laboratories.

More or less simultaneously, Jim Peebles, an astrophysicist part of the research group of Robert Dicke at Princeton University, New Jersey, was working on a model to explain nucleosynthesis in the early universe. Taking into account today's large mass fraction of hydrogen, Peebles concluded that the early universe must have contained radiation of sufficiently high energy to disintegrate nuclei as soon as they were formed. If not, a large part of the hydrogen would have been fused into helium, resulting in a lower hydrogen mass fraction in favour of a larger helium mass fraction in today's universe. Peebles calculated that this radiation would have cooled down with the expansion of the universe to a present temperature of about $10\ \mathrm{K}$. Without realizing it at first, Peebles thereby provided an explanation for the mystery that Penzias and Wilson were facing.

The discovery of the cosmic microwave background was a breakthrough favouring the hot Big Bang model of the universe over the Steady State model, as the existence of the cosmic microwave background could not be explained using the latter. In 1978, Penzias and Wilson were awarded the Nobel Prize for Physics for their remarkable discovery.

4.3.2 Origin

The cosmic microwave background is the afterglow of a time far back when the universe was a hot dense opaque soup of matter and radiation particles. The average photon energy was large enough to set any electron free from its nucleus and matter was therefore completely ionized. The free electrons interacted abundantly with the photons, thereby creating an opaque medium for the photons and achieving a state of thermal equilibrium between matter and radiation, exactly what is required to create blackbody radiation. At a certain moment during the expansion and cooling-down of the universe, the photons were no longer able to keep all matter ionized and free electrons started to combine[2] with nuclei to form atoms. When there were hardly any free electrons left to interact with, matter and radiation decoupled and the universe became transparent for the photons, leaving the blackbody radiation "imprinted" on the universe. Today, about 13.8 billion years later, the CMB's wavelength got stretched out due to the expansion of the universe and has therefore "redshifted" to the microwave part of the electromagnetic spectrum with a peak frequency of about $160\ \mathrm{GHz}$.

The measurements made by Penzias and Wilson at one frequency obviously did not allow to verify whether the cosmic microwave background has the spectrum of blackbody radiation or not. Verifying the hypothesis was long time not possible due to the fact that Earth's atmosphere absorbs radio waves in large portions of the expected spectrum. Only in 1994, it could irrevocably be confirmed by the Cosmic Background Explorer (COBE), a satellite launched in 1989 with the purpose of making measurements outside Earth's atmosphere, that the cosmic microwave background is indeed blackbody radiation with a temperature of $2.725\ \mathrm{K}$. The data obtained with the COBE satellite and the Planck function which best fits the observations are shown in figure 8.

[2] Many textbooks call it the era of "recombination" which is actually not correct as electrons and nuclei had never been combined before.

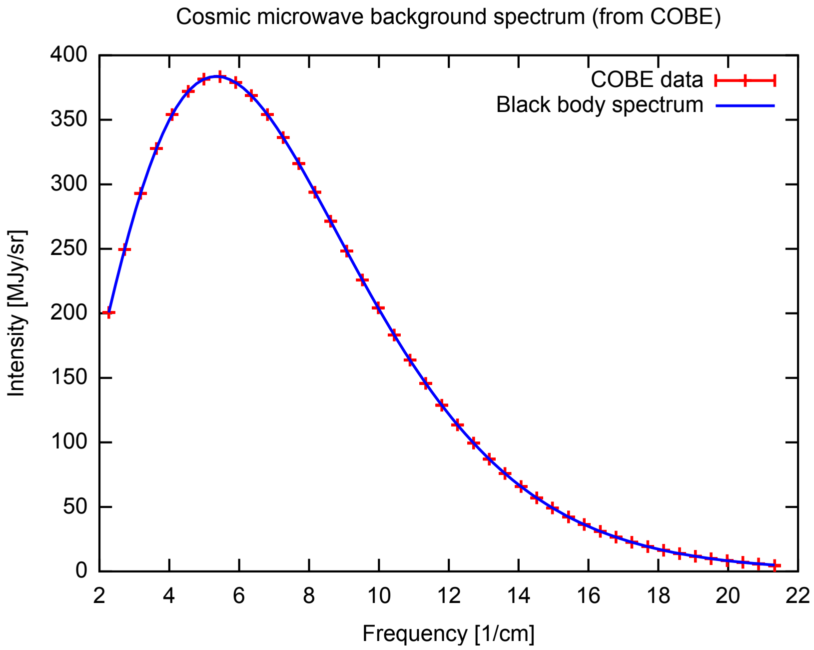

Figure 8: Spectrum of the cosmic microwave background as measured by the COBE satellite. (Credit: Quantum Doughnut, Wikimedia Commons)

4.4 Satellite downlinks

Satellite systems for global positioning, long distance communications, internet, weather monitoring, remote sensing, television broadcasts, etc. use radio waves in the microwave portion of the spectrum for their up- and downlinks from and to Earth based control stations and service subscribers. Well known systems and companies are GPS, GLONASS, Beidou, Galileo, Inmarsat, Iridium, Starlink, Meteosat and Astra to name only a few.

Satellites are either geostationary or orbiting. Geostationary satellites are located in the equatorial plane at a distance of about $35786\ \mathrm{km}$ from Earth's surface, such that the satellites revolve around the Earth at the same pace as the Earth spins around its axis. This makes them appear stationary with respect to Earth. Orbiting satellites on the contrary, rise and set at regular intervals. Given an orbital period of roughly 1.5 to 2 hours for low orbit satellites, they transit across the sky at a rate of about $3°$ to $4°$ per second.

4.5 Atmospheric emission

Earth's atmosphere behaves like an absorbing/emitting foreground cloud when observing extra-terrestrial sources. In the range between $300\ \mathrm{MHz}$ and $300\ \mathrm{GHz}$, mainly oxygen and water (vapor) molecules absorb and re-emit radio waves. Both molecules are abundantly present and therefore significantly contribute to the atmospheric emission. The effect is more pronounced above $15\ \mathrm{GHz}$ as shown in figure 9 due to the presence of several strong absorption lines.

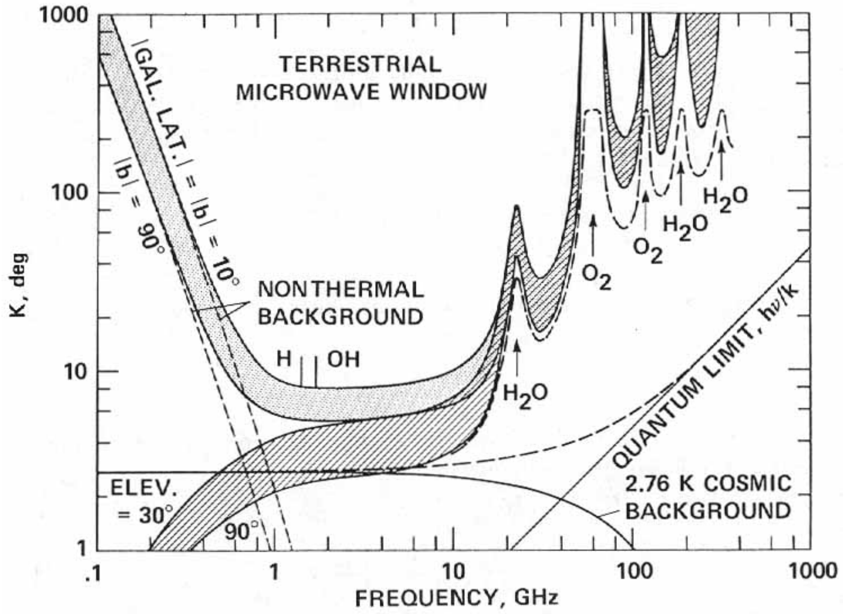

Figure 9: Terrestrial microwave window. Atmospheric water vapour and oxygen degrade the upper end of the microwave window for receivers on Earth's surface and raise the temperature in the lower portion of the window. (Credit: NASA)

Water is not as evenly distributed as oxygen and also exists in liquid or solid form in clouds. This leads to variations in atmospheric emission, both in time and direction. At high altitudes, the layer of overhead atmosphere is obviously smaller than at ground level and on cold cloudless days, the air contains less water in various forms. This is beneficial for observing extra-terrestrial sources.

The equivalent temperature of Earth's atmosphere T_{atm} depends on the elevation angle ϵ of the antenna as the lower the elevation angle, the larger the column of air in the antenna's line of sight. The lowest temperature T_{zen} is measured when the antenna points straight upward to the zenith, yielding the relation:

$$T_{atm} = \frac{T_{zen}}{sin\ \epsilon}$$

It is convenient to introduce the dimensionless quantity air mass $A = 1/sin\ \epsilon$, which in essence is a measure for the column of air in the line of sight of the antenna compared to the column of air in vertical direction:

$$T_{atm} = A\, T_{zen}$$

The difference between 2 voltage measurements V_1 and V_2, taken at different elevation angles ϵ_1 and ϵ_2 with corresponding air masses A_1 and A_2, is given by:

$$V_1 - V_2 = V_K\, (A_1 - A_2)\, T_{zen}$$

Reworking this equation yields:

$$T_{zen} = \frac{1}{V_K}\left(\frac{V_1 - V_2}{A_1 - A_2}\right) \tag{17}$$

The accuracy of T_{zen} is increased by taking measurements at more than 2 elevation angles, fitting a line to the data (e.g. by the least squares method) and determining the slope of the fit.

4.6 Ground spill-over

All practical directional antennae are characterized by a radiation pattern which consists of a main lobe and smaller side lobes at increasingly wider angles from the main one. The aim is to select an antenna with minimal side lobes. Corrugated and flared horn antennas perform well in this respect at the expense of a lower effective aperture.

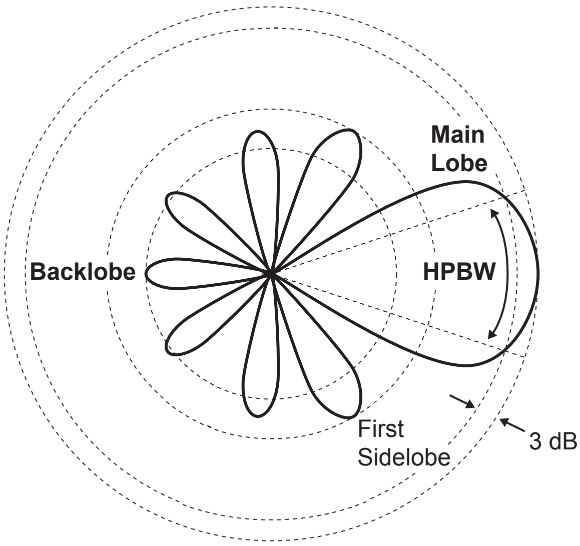

Figure 10: Polar plot of the radiation pattern of a directional antenna. The half power beam width (HPBW) is the sector of the main lobe within which the magnitude decreases to $50\ \%$ (or $3\ \mathrm{dB}$) of the maximum. (Credit: Offaperry, Wikimedia Commons)

Even at higher antenna elevation angles, ground radiation from trees, buildings and other structures is potentially picked-up by the side lobes, resulting in a higher measured equivalent temperature than actually received from the source under observation in the main lobe. This effect is known as ground spill-over.

4.7 Receiver noise

Thermal noise is present in all electrical circuits and is caused by the thermal agitation of electrons inside electrical conductors. Rewriting equation (11) yields an expression for the equivalent temperature T_{rcv}:

$$\boxed{T_{rcv} = \frac{V_H}{V_K} - T_H} \tag{18}$$

Obviously, the thermal agitation and therefore also the amount of thermal noise depends on the temperature of the electronic circuits in the receiver, which is why professional radio telescope receivers are cooled to cryogenic temperatures. Thermal stabilization ensures that the noise figure does not change too much between calibration and observation due to temperature fluctuations.

5 Ground based CMB detection

5.1 Power contributions

The different power contributions picked-up by a radio telescope are all characterized by their equivalent temperature and in general consist of the radio source under observation, galactic emission, the cosmic microwave background, satellites entering the antenna beam, atmospheric emission, ground spill-over and receiver noise. The output voltage V_{out} of the power detector is then proportional to the sum of all the equivalent temperatures. When observing the cosmic microwave background from Earth's surface, the contribution of interest $T_A = T_{cmb}$ and equation (7) expands to:

$$V_{out} = V_K \left(T_{cmb} + \underbrace{T_{src} + T_{gal} + T_{sat} + A\,T_{zen} + T_{gnd} + T_{rcv}}_{T_{sys}}\right) \tag{19}$$

Remember that for a source which uniformly surrounds the antenna like the cosmic microwave background, the antenna temperature is a direct measure of the source's brightness temperature. T_{cmb} is therefore not only the CMB's equivalent temperature but also its brightness temperature. And as the cosmic microwave background is blackbody radiation, it also corresponds to its thermodynamic temperature.

5.2 Unwanted contributions

Discrete radio sources are point-like sources for antennae with relatively large beam widths such as the horn antennae generally used for CMB experiments. They are rather weak and their contribution to the overall measured equivalent temperature is therefore very small. Additionally, their position in the sky is well known which makes them avoidable during observation runs.

As illustrated by figure 6, observations near the galactic poles will result in a lower noise level caused by galactic radiation. In any case, it is generally of an order of magnitude of $10\,\mathrm{mK}$ at $10\,\mathrm{GHz}$ and therefore of little concern when one wishes to detect a source in the order of a few Kelvin such as the cosmic microwave background.

If the cosmic microwave background is blackbody radiation with a temperature of $2.725\ \mathrm{K}$, equation (1) yields that the peak of the spectrum is located close to $160\ \mathrm{GHz}$. To obtain the best signal to noise ratio, the cosmic microwave background would best be observed at that frequency but unfortunately, it falls outside the terrestrial microwave window shown in figures 2 and 9.

To counteract ground spill-over, it is beneficial to use corrugated or flared horn antennae. These types of antennae namely have smaller side lobes at the expense of a lower effective aperture. Another possibility is to shield the side lobes from ground radiation by using radiation screens made of a metallic mesh as illustrated in figure 11.

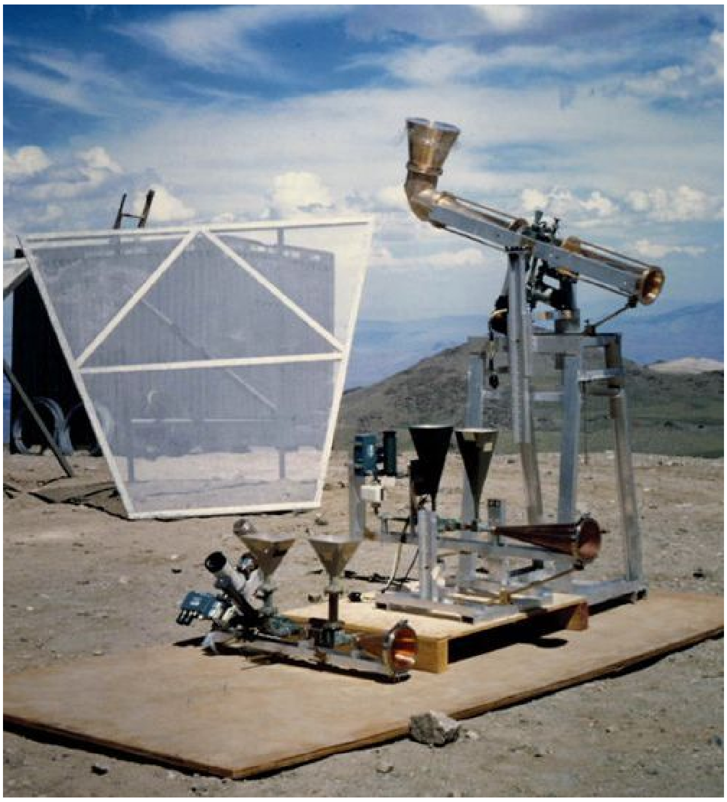

Figure 11: Ground screen (trapezoidal white mesh) shielding the horn antenna from radiation emitted or reflected by the background shed. (Credit: Stokes, Partridge and Wilkinson)

5.3 Corrected brightness temperature

Based on equation (19) and assuming that effective observational measures are in place such that $T_{src} = T_{gal} = T_{sat} = T_{gnd} \approx 0$, the CMB brightness temperature T_{cmb} is given by:

$$T_{cmb} = \frac{V_{out}}{V_K} - A\,T_{zen} - T_{rcv}$$

The factor V_K is derivable from either equation (10) or equation (12) while T_{zen} and T_{rcv} are obtainable from respectively equations (17) and (18).

The measured brightness temperature of the cosmic microwave background is strictly speaking not its true brightness temperature due to the fact that Earth's atmosphere acts as an absorbing and emitting foreground cloud for the cosmic microwave background.

If T_{src} is the temperature of a source which is observed through an absorbing and emitting foreground cloud of temperature T_{cld} with optical depth τ, then the observed temperature T_{obs} is given by [Par95, p. 116]:

$$T_{obs} = \underbrace{T_{src}\, e^{-\tau}}_{\text{absorption}} - \underbrace{T_{cld}\,(1 - e^{-\tau})}_{\text{emission}} \tag{20}$$

The right hand term of equation (20) represents the atmospheric emission previously discussed which contributes to the total measured equivalent temperature. There is however also an attenuation part to be taken into account. The left hand term of equation (20) shows that a correction factor e^{τ} needs to be applied to obtain the CMB's true brightness temperature T_{CMB} from the measured brightness temperature T_{cmb}:

$$\boxed{T_{CMB} = T_{cmb}\, e^{\tau}}$$

A good approximation for our atmosphere's optical depth τ is obtainable from the right hand term of equation (20), applied to the measured zenith temperature T_{zen}:

$$T_{zen} = T_{cld}\,(1 - e^{-\tau})$$

If $\tau \ll 1$, which is the case for Earth's atmosphere, $e^{-\tau}$ is approximated by $1 - \tau$ and hence:

$$T_{zen} = T_{cld}\,\tau$$

Under the assumption that the average temperature of the atmosphere is $250\ \mathrm{K}$, this yields:

$$\boxed{\tau = \frac{T_{zen}}{250\ \mathrm{K}}}$$

For a typical zenith temperature T_{zen} of $1.25\ \mathrm{K}$, the atmosphere's optical depth τ would be 0.005 .

5.4 Summary of prerequisites

To positively observe the cosmic microwave background from Earth's surface, it is of paramount importance to select an observing frequency which falls into the terrestrial microwave window. Even then, the contribution from Earth's atmosphere to the total measured power needs to be determined and subtracted from the observational result. The same applies for receiver noise. Furthermore, unwanted contributions and adverse factors have to be kept as small as practically possible by

- avoiding that discrete radio sources or orbiting satellites enter the antenna beam; and/or
- observing an area of the sky far away from the galactic plane to minimize the galactic radiation contribution; and/or
- observing an area of the sky away from the equatorial plane to avoid geostationary satellites; and/or
- selecting an isolated observation site at a high altitude to minimize the amount of overhead atmosphere and consequently its influence; and/or
- observing on a dry cold cloudless day to limit the radiation contribution and noise from water molecules; and/or
- using ground screens to shield the antenna from ground radiation; and/or
- using a corrugated or flared horn antenna to minimize the side lobes and consequently ground spill-over into the side lobes; and/or
- cooling the receiver to minimize receiver noise; and/or
- stabilizing the receiver's internal temperature to avoid gain fluctuations.

References

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